

Magnetic phases of $t - J$ model on triangular lattice

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We study the magnetic properties of the $t - J$ model on triangular lattice in light of the recently discovered superconductivity in Na_xCoO_2 system. We formulate the problem in the Schwinger Boson - slave Fermion scheme and proposed a sound mean field ansatz(canting ansatz) for the RVB order parameters. Working with the canting ansatz, we map out the temperature-doping phase diagram of the model for both sign of the hopping term. We find the prediction of the $t - J$ model differ drastically from that of earlier LSDA calculation and there is large doping range in which the system show zero net magnetization, rather than saturated magnetization as predicted in the LSDA calculation. We show the result of LSDA is unreliable in the strong coupling regime due to its neglect of electron correlation. We find the spin Berry phase play a vital role in this geometrically frustrated system and the various states in the phases diagram are characterized(and distinguished) by their respective spin Berry phase, rather than any Landau-like order parameter related to broken symmetry. We find the spin Berry phase is responsible for the qualitative difference in the low energy excitation spectrum of the various states of the phase diagram. We argue the phase boundary in the mean field phase diagram may serve as the first explicit and realistic example for phase transition between states with different quantum orders which in our case is nothing but the spin Berry phase. We also find an exotic state with nonzero spin chirality but no spin ordering is stable in a large temperature and doping range and find this state support a nonzero staggered current loop in the bulk of the system. We propose to study this exotic state in the hole doped $\text{Na}_{1-x}\text{TiO}_2$ system. As a by-product of this study, we also achieve an improvement over earlier Schwinger Boson mean

field theory of the triangular antiferromagnet by obtaining two gapless spinon modes. We find the small gap of the third spinon mode is caused by quantum fluctuation and our theory reproduce the linear spin wave theory(LSWT) result in the semiclassical limit.

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I. INTRODUCTION

The recent discovery of superconductivity below $T_c \simeq 4.5K$ in $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$ ($x \simeq 0.35, y \simeq 1.3$)¹ has aroused much research interest. This material constitutes another example of superconducting transition metal oxides with layered structure besides the cuprates. The similarity between the two systems make it possible that the study of the new superconductor may shed light on those difficult problems in the study of the cuprates. In Na_xCoO_2 material, the Co^{4+} ion possess a half spin and play a similar role as the Cu^{2+} ion in cuprates. However, the Co^{4+} in Na_xCoO_2 form a triangular lattice rather than square lattice as in the cuprates. From theoretical point of view, triangular lattice system is even more interesting since it is intrinsically frustrated.

Shortly after the discovery of superconductivity several theoretical proposals are raised to address its mechanism²⁻⁴. On the basis of the mean field analysis on $t - J$ model, a $d + id'$ pairing state is proposed by Barskran², while Tanaka et al proposed a triplet pairing state based on symmetry consideration³. Experimentally, the pairing symmetry is still under debate⁵⁻⁷. Besides superconductivity, this system also exhibit nontrivial magnetic properties. In particular, a phase transition at $22 K$ from paramagnetic to weak ferromagnetic state is reported in $\text{Na}_{0.75}\text{CoO}_2$ ⁸. Experience in the study of cuprates tell us that the magnetic degree of freedom may play an important role in the low energy physics of this strongly correlated system. Especially, it is important to find out the relation between the magnetic properties and the superconductivity. Thus a comprehensive understanding of the magnetic properties of this system is valuable. In this respect, Singh calculated the magnetic properties of the system in the LSDA scheme⁹ and found that the carrier in the conduction band is always fully polarized, even in the half filled case. This result is inconsistent with the result of the strong coupling analysis based on $t - J$ model. In the LSDA-like weak coupling treatment, the tendency of the system toward ferromagnet state is exaggerated due to the absence of electron correlation(the electron avoid each other only through the Pauli exclusion principle which make the ferromagnetic ordering of the electron spin energetically favorable). In technical terms, the Stoner factor $1 - UN(E_F)$ overestimate the ferromagnetic instability in the strongly correlated regime. In fact, as U increase, the density of state at the Fermi energy is also depressed. Especially, at the half filled case, $N(E_F)$ may reduce to zero due to the Mott insulator physics. Thus a through study of the magnetic properties of the system in the strong coupling regime is deserved.

In this paper, we study the magnetic properties of the system within the $t - J$ model perspective. Complementary to the weak coupling picture, in the strong coupling regime, it is the reduction of kinetic energy rather than the

potential energy that dominate the low energy physics and thus determine the magnetic properties since the no double occupancy constraints in the $t - J$ model already take into account of the effect of the potential energy in the zeroth order. Here the kinetic energy contains contribution from both real charge transport of doped holes and the virtual hopping process that lead to spin exchange. The real and virtual part of the kinetic energy are competing with each other. The real kinetic energy favor ferromagnetic alignment of the spin due to the Nagaoka physics while the virtual kinetic energy or the superexchange favor antiferromagnetic alignment of spin since it is blocked by Pauli exclusion principle. At half filling, the real process is depressed and the virtual process dominate. Thus a fully polarized state is obviously unfavorable in this case. In fact, the triangular $t - J$ model show zero net magnetization at the half filling and is generally believed to be in a coplanar 120° state with three sublattices¹⁰⁻¹³(see Figure 1). How dose this state evolve with doping? This is the central problem we want to address in this paper. We find the answer depend crucially on the sign of the hoping term of the $t - J$ model(on a triangular lattice, the particle hole symmetry is broken and the sign of the hopping term is essential). A negative hopping matrix element will induce a π -flux around each triangular plaquette of the lattice and frustrate the motion of the charge carrier. The charge carrier will also see another flux due to the spin Berry phase in a noncollinear spin background. The interplay between the two result in a complex doping dependence of the magnetic properties found in this work. Although the hopping term in the Na_xCoO_2 system is positive, we will consider both positive and negative sign case in our paper for general interest.

In this paper, we treat the $t - J$ model in the slave Fermion - Schwinger Boson formalism since it is convenient for the discussion of magnetic properties. At half filling, the spin system is in a coplanar state with three sublattices. The spin in the three sublattices lay symmetrically on their common plane with a 120° angle between each other. From this state, the most natural way for the system to obtain a net magnetization is to tilt the spin of the three sublattices symmetrically above their common plane. This is supported by a unrestricted search of the semiclassical energy functional. The state so obtained is called a canting state. With this picture in mind, we propose a sound ansatz for the mean field RVB order parameter for the spins. We call the mean field ansatz constructed in this way the canting ansatz. Working with the canting ansatz, we map out the whole doping-temperature phase diagram of the $t - J$ model for both sign of the hopping term. The main result result of this paper can be summarized as follow.

We find there is large doping range in which the system show zero net magnetization, rather than saturated magnetization as predicted in the LSDA calculation. Thus the LSDA result is unreliable in the strong coupling regime.

We find the spin Berry phase play a vital role in this geometrically frustrated system. We find the various states in the phase diagram are characterized(and distinguished) by their respective spin Berry phase, rather than any Landau-like order parameter related to broken symmetry. We also find the spin Berry phase is responsible for the qualitative difference in the low energy excitation spectrum of the various states in the phase diagram. We argue that the phase boundary in the mean field phase diagram are true phase transition lines and may serve as the first explicit and realistic example for phase transition between states with different quantum orders¹⁴. We argue the spin Berry phase provide a first realistic example for the concept of quantum order.

We find there exists an exotic state with nonzero spin chirality but no spin ordering in a large temperature and doping range. We find this exotic state support nonzero staggered current loop in the bulk of the system and propose to study this exotic state in the hole doped $\text{Na}_{1-x}\text{TiO}_2$ system.

As a by-product of this study, we also achieve an improvement over earlier Schwinger Boson mean field theory on

the triangular antiferromagnet. The canting ansatz proposed in this paper lead to two branches of gapless spinon modes. We find the small gap of the third spinon mode is caused by quantum fluctuation and our theory reproduce the linear spin wave theory result in the semiclassical limit.

Finally, we find our general phase diagram is consistent with experimental result on $\text{Na}_{1-x}\text{CoO}_2$ system and a singlet pairing picture may be more relevant in the small doping regime.

The paper is organized as follows. In section II, the Schwinger Boson-slave Fermion representation for the $t - J$ model and our working ansatz, the canting ansatz are introduced. The phase diagram is presented in section III. In section IV, we present the result for the doping dependence of magnetization and excitation spectrum. In the concluding section IV, relevance of our result to experiments are discussed.

II. THE CANTING ANSATZ

Our starting point is the $t - J$ model on triangular lattice,

$$H = -t \sum_{\langle i,j \rangle, \sigma} (\hat{C}_{i\sigma}^\dagger \hat{C}_{j\sigma} + H.c.) + J \sum_{\langle i,j \rangle} \hat{S}_i \hat{S}_j \quad (1)$$

in which $\hat{C}_{i\sigma}$ satisfy the no double occupation constraint $\sum_{\sigma} \hat{C}_{i\sigma}^\dagger \hat{C}_{i\sigma} \leq 1$. $\sum_{\langle i,j \rangle}$ denotes sum over nearest-neighbouring sites. For the triangular lattice, the sign the t is essential and we will consider both sign of t for theoretical interest, although t is positive in Na_xCoO_2 system.

At half filling, the model reduce to the much studied antiferromagnetic Heisenberg model on a triangular lattice. It is believed that this model posses a coplanar magnetic order with three sublattices at zero temperature¹⁰⁻¹³. In this state, the spins in the three sublattices reside symmetrically on their common plane with 120° angle between each other (see Figure 1). This result can be obtained in the Schwinger-Boson mean field theory of the model^{12,13}. In this formalism, the spin operator \hat{S}_i is represented by Schwinger Boson $b_{i,\alpha}$

$$\hat{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \sigma_{\alpha\beta} b_{i\beta}$$

with the constraint $\sum_{\alpha} b_{i\alpha}^\dagger b_{i\alpha} = 1$ and the Hamiltonian is

$$H_s = \frac{1}{4} J \sum_{\langle i,j \rangle} b_{i\alpha}^\dagger \sigma_{\alpha\beta} b_{i\beta} b_{j\gamma}^\dagger \sigma_{\gamma\delta} b_{j\delta}$$

In the mean field treatment, order parameter $D_{ij} = \langle b_{i\uparrow} b_{j\downarrow} - b_{i\downarrow} b_{j\uparrow} \rangle$, $Q_{ij} = \left\langle \sum_{\alpha} b_{i\alpha}^\dagger b_{j\alpha} \right\rangle$ are introduced to represent the spin correlation in the system. Although D_{ij} and Q_{ij} are themselves spin rotational invariant, by condensing the bosonic spinon $b_{i\alpha}$ we can still break this symmetry and describe magnetic ordered state (note both D_{ij} and Q_{ij} are needed for a nonbipartite lattice to find the absolute minima of the energy functional). The choice of the mean field ansatz D_{ij} and Q_{ij} depends on our understanding of the ground state. The ansatz used in previous theories¹¹⁻¹³ on this problem can reproduce the three sublattice magnetic order, but fails to give three branches of gapless spin wave and are thus not fully satisfactory. In the following, we will determine the mean field ansatz with a semiclassical analysis. When the system is doped, we should introduce another slave particle to represent the hole degree of freedom. In this paper, we use the slave Fermion- Schwinger Boson representation. In this formalism

$$\hat{C}_{i\sigma}^\dagger = f_i b_{i\sigma}^\dagger,$$

in which f_i is Fermionic operator for the holon, and $b_{i\sigma}^\dagger$ is bosonic operator for the spinon just defined. The no double occupation constraint is now expressed as:

$$f_i^\dagger f_i + \sum_{\sigma} b_{i\sigma}^\dagger b_{i\sigma} = 1,$$

and the Hamiltonian (1) is:

$$\begin{aligned} H = & -t \sum_{\langle i,j \rangle} (f_i f_j^\dagger b_{i\sigma}^\dagger b_{j\sigma} + h.c.) + \frac{1}{4} J \sum_{\langle i,j \rangle} b_{i\alpha}^\dagger \sigma_{\alpha\beta} b_{i\beta} b_{j\gamma}^\dagger \sigma_{\gamma\delta} b_{j\delta} \\ & + \sum_i \lambda_i (f_i^\dagger f_i + b_{i\sigma}^\dagger b_{i\sigma} - 1) - \mu \sum_i f_i^\dagger f_i, \end{aligned} \quad (2)$$

in which λ_i is the Lagrangian multipliers to enforce particle number constraint and μ is the chemical potential for the hole. This Hamiltonian can be decoupled into bilinear form with the introduction of the mean field order parameter D_{ij} and Q_{ij} . In principle, we should also include the order parameter $F_{ij} = \langle f_i^\dagger f_j \rangle$ in the doped case. However, since the hole has no dynamics of its own, F_{ij} is proportional to Q_{ij} and is not an independent variable. After the decoupling, we get the mean field Hamiltonian

$$H_s = \sum_{\langle i,j \rangle} \left[(tF_{ij}^* + \frac{J}{4}Q_{ij}^*)b_{i\alpha}^\dagger b_{j\alpha} - \frac{J}{4}D_{ij}^*\epsilon_{\alpha\beta}b_{i\alpha}^\dagger b_{j\beta}^\dagger + H.c. \right] - \mu_b \sum_i b_{i\alpha}^\dagger b_{i\alpha}$$

for spinon and

$$H_h = \sum_{\langle i,j \rangle} \left[tQ_{ij}f_j^\dagger f_i + H.c. \right] - \mu_f \sum_i f_i^\dagger f_i$$

for holon. Here $\epsilon_{\alpha\beta}$ is the antisymmetric tensor. μ_b and μ_f are the spinon and holon chemical potential to enforce the particle number constraints $\langle b_{i\alpha}^\dagger b_{i\alpha} \rangle = 1 - x$ and $\langle f_i^\dagger f_i \rangle = x$, where x is the hole concentration. From this Hamiltonian, we see the spin background affect the hole motion through the order parameter Q_{ij} whose phase is nothing but the spin Berry phase mentioned in the Introduction.

Now we determine the mean field ansatz D_{ij} and Q_{ij} from a semiclassical analysis¹⁵. In the semiclassical limit, we can take $b_{i\alpha}$ as a two component spinor

$$b_i = \sqrt{S} \begin{pmatrix} e^{-i\varphi_i} \cos(\frac{1}{2}\theta_i) \\ \sin(\frac{1}{2}\theta_i) \end{pmatrix} \quad (3)$$

in which θ_i and φ_i are spherical coordinates of the spin S_i . Here an arbitrary gauge phase factor $e^{i\chi_i}$ is omitted. Now let us consider possible magnetization of the system from the coplanar state at half filling. In this parent state, the spins in the three sublattices lay symmetrically in their common plane. Hence the most natural way for the system to obtain a net magnetization is to tilt the spins of the three sublattices symmetrically above their common plane. We call such a state a canting state. In this state, the semiclassical spin on the three sublattices are

$$\begin{aligned} b^A &= \sqrt{S} \begin{pmatrix} \cos(\frac{1}{2}\theta) \\ \sin(\frac{1}{2}\theta) \end{pmatrix} \\ b^B &= \sqrt{S} \begin{pmatrix} e^{-i\frac{2\pi}{3}} \cos(\frac{1}{2}\theta) \\ \sin(\frac{1}{2}\theta) \end{pmatrix} \\ b^C &= \sqrt{S} \begin{pmatrix} e^{-i\frac{4\pi}{3}} \cos(\frac{1}{2}\theta) \\ \sin(\frac{1}{2}\theta) \end{pmatrix}, \end{aligned} \quad (4)$$

where A, B, C stands for the three sublattices. The coplanar state at half filling is a special case of the canting state with $\theta = \frac{\pi}{2}$. This canting ansatz is supported by minimizing the semiclassical energy functional. This energy functional is obtained by substituting (2) into (??) (with \sqrt{S} replaced by $\sqrt{S(1-\delta)}$ in the doped case, where δ is the hole concentration) and then diagonalize the holon Hamiltonian. We performed an unconstraint search on a 12×12 lattice and find the canting state always has the lowest energy. In the semiclassical limit, the mean field order parameter can be readily written down. For the canting state, we have

$$\begin{aligned} D_{AB} &= \langle b_{A\uparrow} b_{B\downarrow} - b_{A\downarrow} b_{B\uparrow} \rangle = D_{BC} e^{i\frac{4}{3}\pi} = D_{CA} e^{-i\frac{4}{3}\pi} = D \\ Q_{AB} &= \left\langle \sum_{\alpha} b_{B\alpha}^\dagger b_{A\alpha} \right\rangle = Q_{BC} = Q_{CA} = Q \end{aligned} \quad (5)$$

To fix the gauge totally, we require D to be real. In this paper, we will study mean field ansatz satisfying (5) and call such a mean field ansatz canting ansatz. In this ansatz, a real order parameter D and a complex order parameter Q are introduced to represent the spin correlation in the system. In particular, the phase of Q equals to one sixth of the solid angle spanned by the spins in the three sublattices and is thus related to the spin chirality and the spin Berry phase of the background spins¹⁶.

The mean field Hamiltonian with the canting ansatz can be diagonalized. Introducing $\Psi_k = (f_k^A \ f_k^B \ f_k^C)^T$ and $\Phi_k = (b_{k\uparrow}^A \ b_{k\uparrow}^B \ b_{k\uparrow}^C \ b_{-k\downarrow}^A \ b_{-k\downarrow}^B \ b_{-k\downarrow}^C)^T$, the Hamiltonian can be written as

$$H_s = \sum_k \Phi_k^\dagger \mathbf{M}_b \Phi_k$$

$$H_h = \sum_k \Psi_k^\dagger \mathbf{M}_h \Psi_k$$

where the expression for the 6×6 matrix \mathbf{M}_b and the 3×3 matrix \mathbf{M}_h are given in the Appendix. Here the momentum sum runs over the reduced Brillouin zone for the three sublattice system. The six branches of spinon dispersions are

$$\omega_n^\pm(k) = \left| -2 \text{Im}(T) \text{Im}(\Gamma_{n,k}) \pm \sqrt{\left[\mu_b + 2 \text{Re}(T) \text{Re}(\Gamma_{n,k}) \right]^2 - \frac{1}{4} \left[D \text{Im}(\Gamma_{n,k}) \right]^2} \right|$$

with $n = 0, 1, 2$. The three branches of holon dispersions are

$$\varepsilon_n(k) = \mu_f + t \text{Re}(Q \Gamma_{n,k})$$

with $n = 0, 1, 2$. Here $T = (-tF + \frac{Q}{4})e^{i\frac{2\pi}{3}}$, $\Gamma_{n,k} = \gamma_k e^{i\frac{2n\pi}{3}}$ and $\gamma_k = \sum_{\delta} e^{ik\delta}$ in which $\delta = (1, 0), (-\frac{1}{2}, \pm\frac{\sqrt{3}}{2})$.

Since $\omega_n^+(k) = \omega_n^-(-k)$, each branch of spinon mode is two-fold degenerate. This is a reflection of the spin rotational symmetry which is unbroken in our treatment.

By solving the self-consistent mean field equations, we can obtain a series of solutions for D and Q . These solutions are characterized by the phase of Q . As we have shown, this phase angle, φ , equals to one sixth of the solid angle spanned by the spins. Since the solid angle is defined modulo 4π and the problem is symmetric under the reflection about the common plane of the spins, we can restrict ourselves to the interval $0 \leq \varphi \leq \frac{\pi}{3}$. In the following, the state with $\varphi = 0$ will be called a collinear state and the state with $\varphi = \frac{\pi}{3}$ will be called the coplanar state. A state with φ between this two limits will be called the canted state. In the next section, we will map out the mean field phase diagram of the model by determining the solution with the lowest free energy.

III. THE PHASE DIAGRAM

A. The $t > 0$ case

The temperature-doping phase diagram for $t/J = 1.5$ is shown in Figure 2. At low doping, the coplanar state has the lowest free energy. Above a critical doping about 0.5 the system transform into the collinear state through a first order phase transition. The canted state is never stable for $t > 0$. This result can be understood as follows. From (1), we see the bare hopping term of the hole (which is $-t$) is renormalized by the spin dependent factor Q . For $t > 0$, the hole will feel a flux of strength $\pi \pm 3\varphi$ around each elementary triangular loop of the lattice (φ is the phase of Q) and is in general frustrated. When $\varphi = \frac{\pi}{3}$, or, when the spin is in the coplanar state, the frustration due to the bare hopping term is released by the spin Berry phase from Q . Thus at low doping, the spins prefer to stay in the coplanar state (this state is of course also favored by the spin exchange energy). At high doping, the situation is reversed and now it is the 'holes' of the nearly filled hole band that are moving. The sign of the hopping term of these 'holes'

is opposite to that of the original hole and thus the frustration due the bare hopping term is absent. Thus at high doping, the spins prefer to align themselves ferromagnetically (the spin exchange energy is suppressed at high doping due to dilution of spins). In the argument raised above, we neglect the effect of the amplitude of the Q . Semiclassical analysis shows that this neglect has no qualitative effect.

At zero temperature, the condensation of the spinon will lead to long range order of spins. In the coplanar state, there are two gapless spinon modes (see discussion in section IV). Both modes condense into the coplanar spin long range order. In the collinear state, there is only one gapless spinon mode. The condensation of this mode give rise to ferromagnetic long range order of the spins. Thus unlike the LSDA result, there is a finite doping range in our phase diagram in which the net magnetization is zero.

At finite temperature, the spin long range order does not exist. The distinction between the coplanar state and the collinear state is now more subtle. On symmetry ground, it is hard to tell apart between the two. In our theory, the

two states are distinguished by their spin Berry phase. The problem here is whether the phase boundary between the coplanar state and the collinear state represent true phase transition or just crossover. We think this phase boundary still represent true phase transition since there is a discontinues change of the spin correlation pattern(embodied in the spin Berry phase) across it. This discontinues change in the spin correlation pattern lead to a discontinues change in the excitation spectrum and thus thermodynamical properties of the system across the phase boundary. As will be shown in the next section, the spin Berry phase is responsible for the qualitative difference in the excitation spectrum of the various states in the phase diagram. In particular, in the coplanar state with $\varphi = \frac{\pi}{3}$, there is always two branches of gapless spinon(the gap due to finite temperature is exponentially small), both of which have linear dispersion. While in the collinear state (or any states with $\varphi \neq \frac{\pi}{3}$), there is only one branch of gapless spinon whose dispersion is quadratic. In other words, the two branches of gapless spinon mode in the coplanar state seems to be protected by some kind of order not related to any broken symmetry. This special kind of the order, which is nothing but the characteristic spin Berry phase of the coplanar state, is broken across the phase boundary between the coplanar state and the collinear state. It is in this sense that we think the phase boundary a true phase transition line. The concept of an order with no broken symmetry is first advocated by Wen who give it the name quantum order¹⁴. The quantum order, as it defined by Wen, describes the structure in the quantum wave function which is beyond the classification on symmetry ground and can be detected by checking the excitation spectrum of the system which is sensitive to the structure of the ground state wave function. In our case, the quantum order is nothing but the spin Berry phase. We think the transition between the coplanar state and the collinear state may serve as the first explicit and realistic example for phase transition between states with different quantum orders. This is the most important finding of this paper. In this regard, it is seems natural to look at the zero temperature transition between the two states also as a transition between different quantum orders rather than the conventional Landau-type phase transition, although the symmetry is really broken in this case.

The realistic value of t/J for Na_xCoO_2 is still unsettled. In our theory, the phase boundary between the coplanar state and the fully polarized state will shift to lower doping with increasing t/J . If our assignment of $t/J = 1.5$ is realistic(ARPES experiment report a similar value¹⁷), then the superconductivity observed around $x \sim 0.35$ is covered totally in the coplanar state in which ferromagnetic fluctuation is suppressed. This implies that the singlet pairing picture may be more relevant for the superconductivity in this system. Furthermore, the ferromagnetic transition observed at 22K for $x \sim 0.75$ ⁸ seems to be also consistent with our phase diagram.

B. The $t < 0$ case

The temperature-doping phase diagram for $t/J = -1.5$ is shown in Fig.3. The phase diagram is much more complex as compared with that of the $t > 0$ case. At zero temperature, the spins cant gradually out of their common plane with increasing doping until a critical doping level, above which the spin return back into the coplanar state through a first order phase transition. At finite temperature, the phase diagram show complex structures. Below a lower critical doping, the zero temperature canted state return back into the coplanar state through a second order phase transition at high temperature. Above the lower critical doping, the canted state transform into the collinear state through a second order phase transition. The coplanar state at high doping also transform into the collinear state at high temperature, but through a first order phase transition. The phase diagram contains both first order and second order phase transition lines with tricritical and quadricritical points join them.

This complex phase diagram structure can also be understood on semiclassical ground. For $t < 0$, the frustration due to the bare hopping term is absent and the kinetic energy of the hole favor a state with $\varphi = 0$, or, the collinear state at low doping. The antiferromagnetic exchange energy act against this tendency. The compromise between the two result in the canted state in which the spin cant gradually out of the coplanar state at half filling. At high doping, the situation is again reversed and now the kinetic energy favors the coplanar state which is also favored by the exchange energy. Below the lower critical doping, the spin canting at zero temperature induced by the hole will eventually be erased by the thermal fluctuation and the spins will return to the coplanar state at high temperature. At higher doping level, the hole system is more robust and it in fact help to enhance the spin correlation against thermal fluctuation. This is quite different from the situation in cuprates. In the triangular lattice, the spin is already frustrated at half filling. Doping a small concentration of holes in fact help to release such geometrical frustration. This special effect will be discussed further in the next section.

At finite temperature, the spin long range order can not exist and the various phases in the phase diagram are characterized by the value of the spin Berry phase. Among these phases, of particular interest is the canted phase which hold a special kind of long range order even at finite temperature. In the canted phase, the time reversal symmetry is spontaneously broken and system posses a nonzero spin chirality. Thus the phase boundary between

the canted phase and other phases are true phase transitions rather than crossovers even in the Landau sense. In the canted phase, the spin chirality will induce a nonzero current of the hole. Since the spin chirality is staggered, the induced current is also staggered. Thus in the canted state, we expect staggered current to flow around each triangular loop. This is the most important characteristic of the canted phase. A detection of such current loop is interesting.

Before closing this subsection we note that although the $t < 0$ case of the model is not directly relevant to the $\text{Na}_{1-x}\text{CoO}_2$ system, it is proposed that it may be appropriate for the hole doped system $\text{Na}_{1-x}\text{TiO}_2$ ⁴.

IV. PHYSICAL OBSERVABLE.

A. The doping dependence of the net magnetization.

At zero temperature, the gapless spinon will condense into magnetic long range order. For the canted state and the collinear state, this condensation will lead to a nonzero net magnetization of the system. For the $t > 0$ case (which is relevant for the $\text{Na}_{1-x}\text{CoO}_2$ system), there is discontinuous jump of the magnetization at the transition point between the coplanar state and the collinear state. For $t < 0$, the magnetization build up gradually with increasing doping as the spin cant out of their common plane and jump to zero when the spin return back into the coplanar state at high doping. These results are shown in Figure 4. We also presents the result for the fraction of the condensed spin (which is a measure of magnitude of sublattice magnetization) in this figure. We see doping a small amount of hole enhance the sublattice magnetization for both $t > 0$ and $t < 0$. The reason for this enhancement is that the doped hole can release partially the geometric frustration inherent of the triangular lattice.

B. The spinon excitation spectrum

In quantum systems, the structures in the ground state wave function often have nontrivial consequences in the excitation spectrum of the system. In fact, the excitation spectrum serve as the most direct way to detect any non-symmetry-breaking related structure (or quantum order) in the ground state wave function of quantum system. In our case, the various phases in the phase diagram are distinguished by their spin Berry phase. Now we discuss the effect of the spin Berry phase on the excitation spectrum of the system.

In the reduced Brillouin Zone of the three sublattice system, there are three branches of nondegenerate spinon modes. These three branches of nondegenerate spinon modes transform into each other under the translation in the momentum space by a reciprocal lattice vector of the reduced Brillouin zone (this can be easily shown since $\gamma_{k+G} = \gamma_k e^{\pm i \frac{2\pi}{3}}$, where G is a reciprocal lattice vector of reduced Brillouin zone). Thus in the extended zone scheme, these three branches of spinon modes can be expressed by a single formula

$$\omega(k) = \left| 2 \text{Im}(T) \text{Im}(\gamma_k) + \sqrt{[\mu_b + 2 \text{Re}(T) \text{Re}(\gamma_k)]^2 - \frac{1}{4} [D \text{Im}(\gamma_k)]^2} \right| \quad (6)$$

In the coplanar state T is real (since $\varphi = \frac{\pi}{3}$) and the spinon spectrum can be further reduced to

$$\omega(k) = \sqrt{[\mu_b + 2T \text{Re}(\gamma_k)]^2 - \frac{1}{4} [D \text{Im}(\gamma_k)]^2}$$

This spectrum is degenerate at the six corners of the Brillouin zone of the triangular lattice ($\gamma_k = 3e^{\pm i \frac{2\pi}{3}}$ at these points). This degeneracy is the most important characteristic of the excitation spectrum of the coplanar state and it is protected by the spin Berry in this state. When φ deviate from $\frac{\pi}{3}$, this degeneracy is gone. At zero temperature, the spinon at these momentums will become gapless and condense into the 120° spin long range order. Thus, at general doping there are two branches of gapless spin wave excitation on the ordered spin background (the six corners of the Brillouin zone correspond to two independent momentums). Since $\gamma_k \sim 3 + uk^2$ for small k , both of the two modes have linear dispersion around the gapless point. Besides these two gapless modes, there is a third minimum of the excitation spectrum at the center of the Brillouin zone. The energy of this mode decrease linearly with decreasing doping but remain finite at half filling as shown Figure 5. The excitation spectrum of the half filled system is shown in

Figure 6 and it is compared with spin wave spectrum obtained in the linear spin wave theory(LSWT)¹⁰. The LSWT predict three branches of gapless spin wave on the three sublattice spin background while our Schwinger Boson mean theory predict only two. The reason for this discrepancy can be seen in the following way. Requiring both the zone corner modes and the zone center to be gapless, we obtain

$$D^2 = 3|Q|^2$$

at half filling, a relation valid for coplanar state in the semiclassical limit($S \rightarrow \infty$ limit). Thus the zone center mode will also become gapless in the semiclassical limit and our theory can reproduce the result of the LSWT in this limit. This analysis also indicate that the gap of the zone center mode is due to quantum fluctuation and may be observable for the spin $\frac{1}{2}$ system. We note previous Schwinger Boson mean field theory on the triangular antiferromagnet(with different mean field ansatz)predict only one gapless mode^{11–13}. Thus our result is an improvement over these earlier ones. At finite doping, the gap due to quantum fluctuation is masked by the much larger gap due to the hole background. However, the gaplessness at the zone corner is still intact as long as the system remain in the coplanar state. This provide a explicit example of protected excitation spectrum due to quantum order. Here the quantum order is nothing but the spin Berry phase.

In the canted state and the collinear state(the collinear state is a special case of the canted state), the degeneracy between the zone corner modes is broken. From (6), we see the splitting between the zone corner modes is $\sqrt{3}\text{Im}(T)$ which increase with increasing canting angle. Especially, in the collinear state in which $D = 0, \varphi = 0$, one of the zone corner mode become degenerate with the massive zone center mode. For general canting angle there are three nondegenerate modes in which only one (at zone corner)become gapless at zero temperature. Since $\text{Im}(T)\text{Im}(\gamma_k) \neq 0$ at the gapless point, the dispersion around it is quadratic. The quadratic dispersion is a characteristic of the ferromagnet spin wave. In our case, canting induce a weak ferromagnetic moment. The gaplessness of the quadratic mode indicate that the long wavelength fluctuation of the weak ferromagnetic moment is unaffected by the hole background. The excitation spectrum for a general canting angle is shown in Figure 7.

V. CONCLUSION

Now let's summarize the results of this paper. In this paper, we have studied the magnetic properties of the $t - J$ model on triangular lattice in light of the recently discovered superconductivity in Na_xCoO_2 system. We formulated the problem in the Schwinger Boson - slave Fermion scheme and proposed a sound mean field ansatz, namely the canting ansatz, for the RVB order parameters of spins. With the canting ansatz, we have mapped out the temperature-doping phase diagram of the model for both sign of the hopping term. We find the prediction of the $t - J$ model differs drastically from that of LSDA calculation. In our theory, there is large doping range in which the system show zero net magnetization, rather than saturated magnetization as predicted in the LSDA calculation. We show the result of LSDA is unreliable in the strong coupling regime due to its neglect of electron correlation.

The most important thing found in this paper is the vital role of the spin Berry phase in this geometrically frustrated system. We find the various phases in the phases diagram are characterized(and distinguished) by their spin Berry phase, rather than any Landau-like order parameter related to broken symmetry. We find the spin Berry phase is responsible for the qualitative difference in the low energy excitation spectrum of the various states in the phase diagram. We argue that the spin Berry phase in this system may provide the first explicit example for the concept of quantum order and the transition between state with different spin Berry phase may serve as the first realistic example for phase transition between states with different quantum orders.

Another interesting thing found in this paper is the existence of an exotic state with nonzero spin chirality but no spin ordering in a large temperature and doping range. We find this exotic state, which break the time reversal symmetry, support nonzero staggered current loop in the bulk of the system. Since this state break a discrete symmetry, its phase boundary with other phases are well defined phase transition in the Landau sense. We propose to study this exotic state in the hole doped $\text{Na}_{1-x}\text{TiO}_2$ system.

As a by-product of this study, we also achieve an improvement over earlier Schwinger Boson mean field theory on the triangular antiferromagnet. The canting ansatz proposed in this paper lead to two branches of gapless spinon modes. We find the small gap of the third spinon mode is caused by quantum fluctuation and our theory can reproduce the LSWT result in the semiclassical limit.

Finally, we find our general phase diagram is consistent with experimental result on Na_xCoO_2 system. We find singlet pairing picture for its superconductivity may be more relevant in the small doping regime.

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Appendix: The spinon and holon Hamiltonian

The spinon Hamiltonian in matrix form is

$$\mathbf{M}_b = \begin{pmatrix} \mathbf{m}_1 & \mathbf{t}^* \\ \mathbf{t} & \mathbf{m}_2 \end{pmatrix}$$

where

$$\mathbf{m}_1 = \begin{pmatrix} \mu_b & x & x^* \\ x^* & \mu_b & x \\ x & x^* & \mu_b \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} \mu_b & y & y^* \\ y^* & \mu_b & y \\ y & y^* & \mu_b \end{pmatrix}, \mathbf{t} = \begin{pmatrix} 0 & z_2 & -z_1^* \\ -z_2^* & 0 & z_0 \\ z_1 & -z_0^* & 0 \end{pmatrix}$$

in which $x = (-tF + \frac{J}{4}Q)\gamma_k$, $y = (-tF^* + \frac{J}{4}Q^*)\gamma_k$, $z_n = \frac{J}{4}D\gamma_k e^{i\frac{2n\pi}{3}}$, $\gamma_k = \sum_{\delta} e^{ik\delta}$. The holon Hamiltonian is

$$\mathbf{M}_h = \begin{pmatrix} \mu_F & w & w^* \\ w^* & \mu_F & w \\ w & w^* & \mu_F \end{pmatrix}$$

in which $w = tQ\gamma_k$.

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FIGURES

FIG. 1. The coplanar spin order at half filling and the canting state proposed in this work.

FIG. 2. Temperature-doping phase diagram for $t/J = 1.5$.

FIG. 3. Temperature-doping phase diagram for $t/J = -1.5$.

FIG. 4. Doping dependence of the net magnetization(a and c) and the sublattice magnetization(b and d).

FIG. 5. Doping dependence of the zone center gap in the coplanar state.

FIG. 6. Spinon dispersion of the triangular antiferromagnet as predicted in this work(a) and the spin wave dispersion predicted by the linear spin wave theory(b)¹⁰.

FIG. 7. Spinon dispersion in the canted state. Note the origin of the Brillouin zone has been moved to the gapless point.

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